Problem 9.41

A 2.00 kg mass has velocity $\vec{v}_2 = \left(2.00\hat{i} - 3.00\hat{j}\right)$ m/s.

A 3.00 kg mass has velocity $\vec{v}_3 = \left(1.00\hat{i} + 6.00\hat{j}\right)$ m/s.

a.) Determine the velocity of the system's *center of mass*.

Although I will add some at the end of this problem, the quick and dirty way to do this is by simply using the relationship:

$$M\vec{v}_{cm} = \sum m_i\vec{v}_i = m_2\vec{v}_2 + m_3\vec{v}_3$$

$$\Rightarrow \quad \left[(2.00\text{ kg}) + (3.00\text{ kg})\right]\vec{v}_{cm} = \left[(2.00\text{ kg})\left(2.00\hat{i} - 3.00\hat{j}\right)\text{m/s}\right] + \left[(3.00\text{ kg})\left(1.00\hat{i} + 6.00\hat{j}\right)\text{m/s}\right]$$

$$\Rightarrow \quad (5.00\text{ kg})\vec{v}_{cm} = \left[\left(4.00\hat{i} - 6.00\hat{j}\right)\text{kg}\bullet\text{m/s}\right] + \left[\left(3.00\hat{i} + 18.0\hat{j}\right)\text{kg}\bullet\text{m/s}\right]$$

$$\Rightarrow \quad \vec{v}_{cm} = \frac{\left[\left(4.00\hat{i} - 6.00\hat{j}\right)\text{kg}\bullet\text{m/s}\right] + \left[\left(3.00\hat{i} + 18.0\hat{j}\right)\text{kg}\bullet\text{m/s}\right]}{(5.00\text{ kg})}$$

$$\Rightarrow \quad \vec{v}_{cm} = \left(1.40\hat{i} + 2.40\hat{j}\right)\text{m/s}$$

b.) Determine the momentum of the system's *center of mass*.

$$M\vec{v}_{cm} = (5.00 \text{ kg})\left(1.40\hat{i} + 2.40\hat{j}\right)\text{m/s}$$
$$= \left(7.00\hat{i} + 12.0\hat{j}\right)\text{kg} \bullet \text{m/s}$$

ADDENDUM: (You are done with the problem. The following is for the nerds.)

To visualize what is really going on in this problem, let's assume that at t=0 the 2.00 kg mass is at (0,0) and the 3.00 kg mass is at (10,0).

i.) What will their subsequent motion look like tracked at one-second intervals?

ii.) Where will the system's *center of mass* be at t = 0?

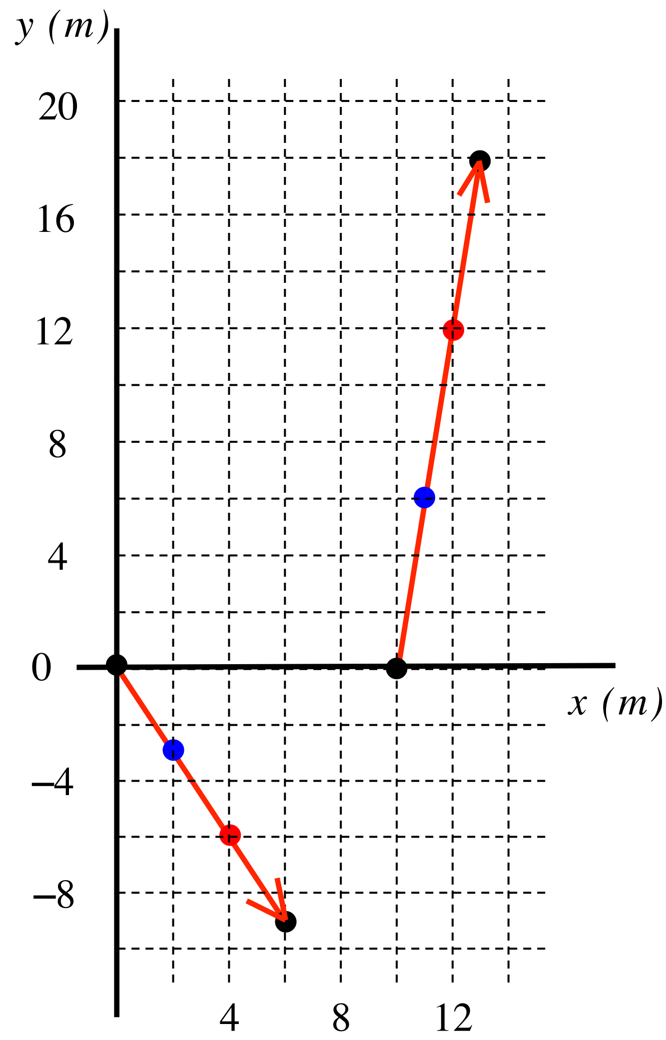
iii.) Through observation, how will the *center of mass* appear to move in one-second intervals?

iv.) Does the apparent motion match up with the calculated *center of mass* velocity?

To visualize what is really going on in this problem, let's assume that at t=0 the 2.00 kg mass is at (0,0) and the 3.00 kg mass is at (0,0).

i.) What will their subsequent motion look like tracked at one-second intervals?

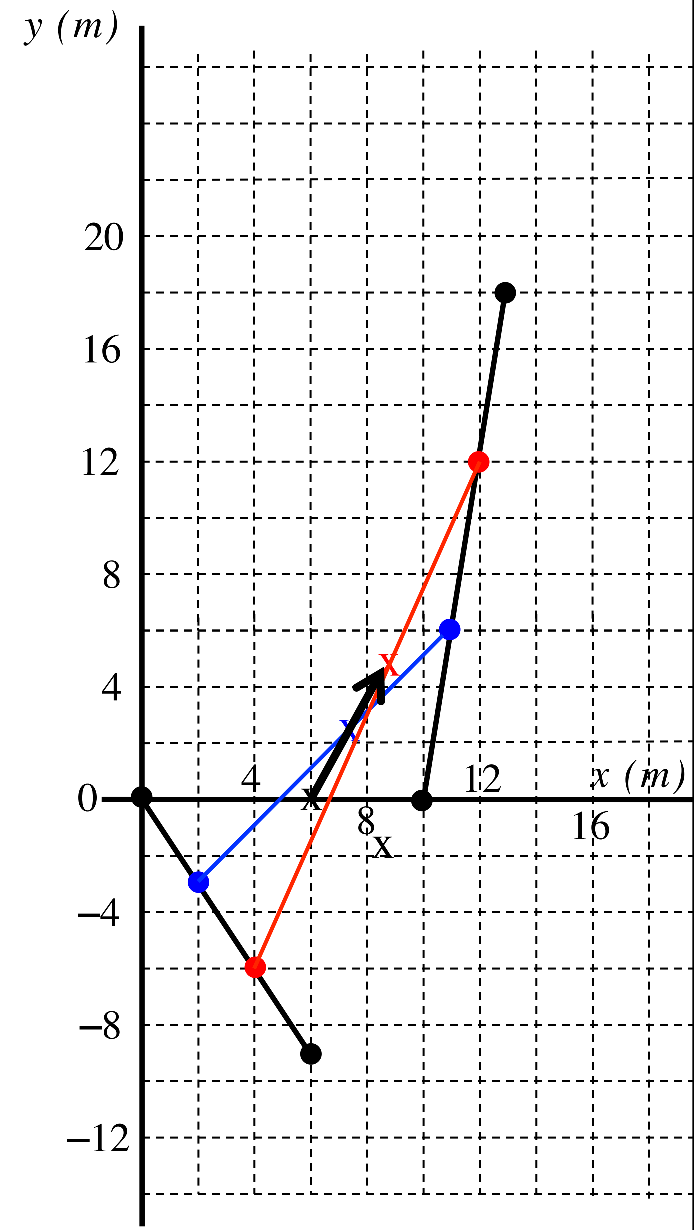
Knowing the position of each at t=0, we can use the velocity relationships to determine where they will be at t=3 seconds. A line between the t=0 and t=3 point for each mass will give us their track. I have, for clarity, put colored points at their t=1 and t=2 second positions.



ii.) The center of mass for this system is at [(2.00)(0)+(3.00)(10)]/(5.00), or at x=6 m. That point is shown with an "x" on the graph.

iii.) Through observation, how will the *center of mass* appear to move in one-second intervals?

The *center of mass* is always on a line between the two masses. At t = 0, it is 6/10 of the way from the left mass. This puts the *center of mass* at (6,0). It also means the *center of mass* will ALWAYS be 6/10 of the way between the two masses. Soooo, look at the graph and the blue dots. These dots represent the mass positions at t=1 second, and 6/10 of the way between those points will be the position of the system's *center of mass* at t=1 second. I've put an "x" at that spot. Similarly for the red dots, and "x" for *that* spot. With that, we can track the direction of the center of mass's motion (it is shown with the black arrow).



iv.) Does the apparent motion match up with the calculated *center of mass* velocity?

The center of mass's velocity was calculated as:

$$\vec{v}_{cm} = \left(1.40\hat{i} + 2.40\hat{j}\right)\text{m/s}$$

Looking at the black arrow, it does appear that it moves in the x-direction around 1.4 units for every 2.4 units it moves in the y-direction. In other words, our graphical evaluation seems to mesh nicely with our mathematical one.

Isn't life wonderful?